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An Inventory Model For Defective Items With Two Different Payment Methods In A Supply Chain System and Inflation

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Abstract

In business, trade credits can be considered as a type of price discount. In this paper, the supplier offers full trade credit to his retailer and the retailer in turn offers partial trade credit to market customers. At the end of the trade credit period, the retailer considers two different payment methods to pay off the loan. In one type of payment method, the retailer settles the account for all units sold, keeps the profits for other use, and starts paying interest charges on the unpaid balance. In another payment method, the retailer pays the supplier the total amount in the interest-bearing account and then starts paying off the amount owed to the supplier whenever the retailer has money obtained from sales.

An inventory model is developed for defective items under the effect of inflation and the time value of money, where demand is a deterministic function of selling price and advertisement cost. A certain fraction of purchased items are defective. These non-conforming items are reworked or refunded if they reach the customer. The model considers a finite replenishment rate under a progressive payment scheme within the cycle time. As a particular case, results of a perfect system (i.e., a system without defective items) are also obtained. The optimal solution is illustrated with numerical examples, and the effect of parameter changes on total cost is graphically presented.

Keywords: Imperfect production, defective item, partial trade credit, inflation, supply chain.
Mathematics Subject Classification: 90B05

1 Introduction

Inflation plays an essential role in determining optimal order policies and influences product demand. As inflation increases, the value of money decreases, eroding the future worth of saving and forces one for more current spending.Usually, these spending are on peripherals and luxury items that gives rise to demand of those items.As a result, the effect of inflation and time value of the money cannot be ignored for determining the optimal inventory policy.as mentioned above, inflation has a major effect on the demand of the goods, especially for fashionable goods for middle and higher income groups. The concept of the inflation should be considered especially for long term investment and forecasting.

The economic order quantity(EOQ) model is a simple mathematical model to deal with inventory management issues in a supply chain. It is considered to be one of the most popular inventory control models used in the industry. Perishable products are commonly found in commerce and industry. Sometimes the rate of deterioration is too low, for items such as steel, hardware, glassware and toys, to cause consideration of deterioration in the deterioration of economic lot sizes. However, some items have a significant rate of deterioration, such as fruits, fresh fishes,perfumes, alcohol, gasoline and photographic films that deteriorate rapidly overtime, which can not be ignored in the decision making process of ordering lot size.

Generally,high selling price of an item affects the demand, which in turn affects the decisions about production and inventory policies.The advertising by the sales team is one of the most important factors used to increase the retailer's profit in modern marketing system. The purpose of the advertisement is to enhance potential customer's responses to a business organization. In general, this strategy is only to sell more items in a short time. Increase in the advertising intensity not only increases the probability of successful marketing targets but also the demand from the customers. Therefore, the more investment in advertising gives more profits for the company. In this direction, our model encourage the retailers to consider the demand as an increasing function of advertising parameter with decreasing value of selling price.

Trade finance signifies financing for trade, and it concerns both domestic and international trade transactions. A trade transaction requires a seller of goods and services as well as a buyer. Various intermediaries such as banks and financial institutions can facilitate these transactions by financing the trade. The trade credit produces three advantages to the supplier, firstly it helps to attract new customer as it can be considered some sort of loan. Secondly, it helps in the bulk sale of goods. The existence of credit

periodservestoreducesthecostofholdingstocktotheuser,becauseitreducesthe
amount of capital invested in stock for the duration of the credit period. Thirdly, it may be applied as an alternative to price discount because it does not provoke competitors to reduce their prices and thus introduce lasting price reductions.
Popular methods of payment used in international trade include:
1. cash with order(CWO)-the buyers pay cash when he places an order.
2. cash on delivery(COD)-the buyer pays cash when the goods are delivered.
documentary credit(L/C)-a Letter of credit (L/C) is used; gives the seller two guarantees that the payment will be made by the buyer:one guarantee from the buyer's bank and another from the seller's bank.
bills for collection(B/E or D/C) -here a Bill of Exchange (B/E)is used; or documentary collection (D/C) is a transaction whereby the exporter entrusts the collection of the payment for a sale to its bank (remitting bank), which sends the documents that its buyer needs to the importers bank (collecting bank), with instructions to release the documents to the buyer for payment.
open account-this method can be used by business partners who trust each other;the two partners need to have their accounts with the banks that are correspondent banks.
Methods of payment: Cash in Advance (Prepayment) Documentary Collections Letters of Credit Open Account Combining Methods of Payment Summary Resources Activities Assessment.
At the end of the trade credit period, some retailers keep their profits for emergency of other use rather than paying off the loan while some retailers will pay off the amount owed to the supplier whenever they have money obtained from sales. That is, the retailer has two possible methods to pay off the loan based on his need. To the best of our knowledge, this research is first to incorporate both two echelon trade credit and two payment methods in a supply chain with perishable items. In one payment method, the retailer pays off all units sold and keeps the profits for other uses. In another payment method, the retailer pays the supplier the total amount in bank account and then starts paying off the amount owed to the supplier whenever the retailer has money obtained from sales. Under these conditions, we intend to develop EOQ models with perishable items. Mathematical expressions are developed to find optimal replenishment time. [1]Annadurai and Uthayakumar formulated an inventory model under

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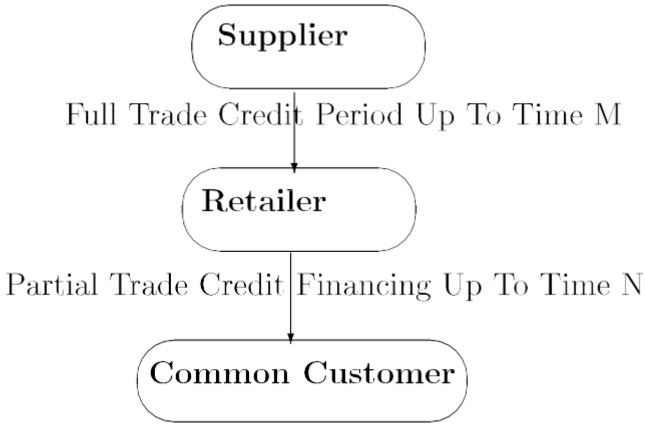
Figure 1: Two - LevelTradeCreditPolicyWithPartialTradeCreditFinancingTo Common Customers



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two levels of credit policy for deteriorating items by assuming the demand is a function of credit period offered by the retailer to the customer.[9]Thangam and Uthayakumar implemented two different payment methods for the retailer to pay off the loan to the supplier under two echelon trade credit scenario. [12]Vandana and Sharma developed an inventory model for deteriorating items with nonlinear demand rate, under the condition of permissible delay in payments, where the suppliers provided permissible delay in payments to the retailers.[3]Jui et al. discussed with a deterministic order level inventory model for deteriorating items with finite warehouse capacity and addresses the conditions of permissible delay in payments. [11]Uthayakumar and Palanivel examined the model for determining the optimal cycle length, optimal production length and optimal quantity for imperfect production process were developed where delay in payments is allowed.[2]Annadurai and Uthayakumar formulated EOQ model for deteriorating items with stock dependent demand under permissible delay in payments.[8]Singh and Singh considered in most of the classical inventory models is constant, while in most of the practical cases the demand changes with time.[10]Thangam and Uthayakumar developed an inventory model for deteriorating items under inflationary conditions using a discounted cash flow approach over a finite planning horizon.[4]Sharmila and Uthayakumar examined the partial trade credit financing in a supply chain by EOQ-based model for decomposing items together with shortages. [5]Sharmila and Uthayakumar presented fuzzy inventory model for deteriorating items with shortages under fully backlogged condition.[6]Sharmila and Uthayakumar presented a mathematical model of an inventory system in which demand depending upon stock level and time with various value of β. It gives more flexibility of the demand pattern and more general tot he study done so far with the condition to minimize the total cost of the system.[7]Sharmila and Uthayakumar considered a continuous inventory model with three rates of production rate under stock and time dependent demand for time varying deterioration rate with shortages.[13]Vijayashree and Uthayakumar presented the problem of a vendor-buyer integrated production inventory model for two stage supply chain under investment for quality improvement.[14]Vijayashree and Uthayakumar presented an integrated a single vendor and a single buyer inventory model in order to minimize the sum of the ordering cost/setup cost, holding cost and crashing cost by simultaneously optimizing the optimal order quantity, lead time and number of deliveries.[15]Vijayashree and

Uthayakumar purposed of this paper is to present the vendor buyer integrated inventory model with lead time reduction for non defective and defective items under investment for quality improvement.

2 Notations and Assumptions

The following notations and assumptions are used through out this paper

2.1 Notations

t_1	duration of the replenishment rate
T	the length of the inventory cycle
$I_1(t)$	the inventory level at time t,$0 \leq t \leq t_1$
$I_2(t)$	the inventory level at time t,$t_1 \leq t \leq T$
R	the constant supply rate of finished goods by the supplier to the retailer
R_1	total number of defective items
μ	the scaling parameter for defective items where defective items $=\mu R^{\delta}$, $\mu > 0$ and $0 < \delta \leq 1$
Q	the order size per cycle for both scenario I and II
A	the ordering cost per cycle
C_1	the holding cost(excluding interest charges)per unit per unit time
P	the purchasing cost per unit
C_r	the rework cost for the defective item(per Unit)
C_{rf}	the refunded cost for the defective item(per unit)
I_e	the rate of interest earned due to financing inventory
r_1	the discount rate which represents the time value of money
f	the inflation rate
r	the net discount rate of inflation i.e.$r = r_1 - f$
TC	the total cost of the system

2.2 Assumptions

1. A single item is considered over an infinite planning horizon.

2. The demand rate D is a deterministic function of selling price s and advertisement cost A_c per unit of item i.e. $D(A_C, s) = A_c^{\eta} a s^{-b}$, $a > 0, b > 1, 0 \leq \eta < 1$. a is the scaling factor , b is the index of price elasticity and η is the shape parameter

3. The replenishment takes place at finite rate

4. The permissible delay in payment is offered by the supplier to the retailer

5. M is greater than N

6. The retailer has two possible payment methods at the end of trade credit period. One is that he keeps his profits for other activities rather than paying off the loan. The other is that he pays off the amount owed to the supplier whenever he has money obtained from sales

7. The retailer just offers the partial trade credit to his customer.Hence his customer must pay off the remaining balance at the end of the trade credit offered by the retailer. That is, the retailer can accumulate interest from his customer payment with trade I_e.

8. Shortages are not permitted

9. The effects of inflation and time value of money are considered

10. The lead time is zero

11. The imperfect(defective) items are considered

3 Mathematical Formulation

In the selling environment, to maintain the goodwill of the firm, the defective items, R_1 must be reworked or refunded, if those are sold to the customer. Considering these situations, we investigate the two different scenarios as follows.

3.1 ScenarioI

Q units are purchased items including the defective ones and sold to the customer at the rate of D units as good units and later R_1 defective units are refunded from the customer with penalty at a cost of $C_{rf}(> s)$ per units.

3.2 Scenario II

Q units are purchased and R_1 purchased defective units are spotted just after purchasing, repaired against the cost of C_r per units and sold as good items to the customer.

3.3 Model Description

The inventory system developed as follows: the inventory cycle starts at $t = 0$ with zero inventory and increase at a rate R and also simultaneously decreases at a rate D up-to the time t_1, and the inventory level is decreasing only due to demand rate in the interval $[t_1, T]$, which finally reaches zero level at time T. Based on the above

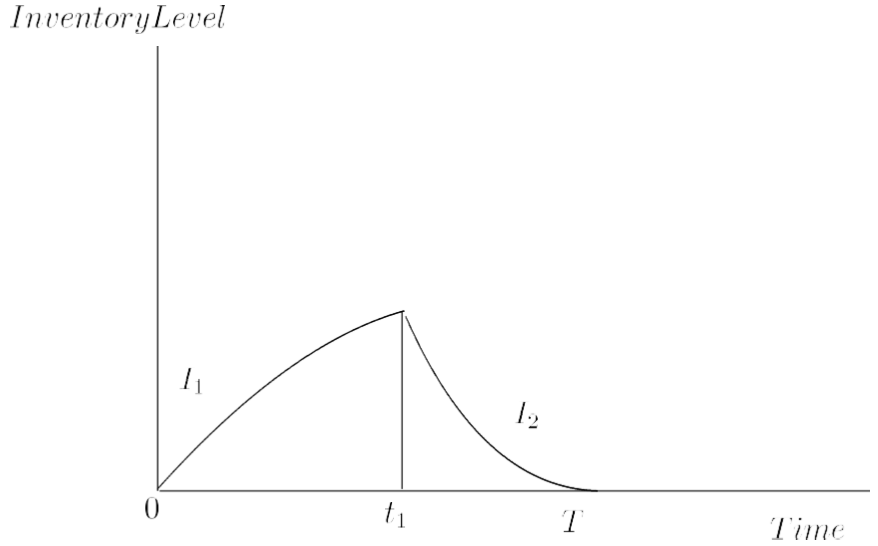

Figure 2: Graphical Representation of The Inventory System

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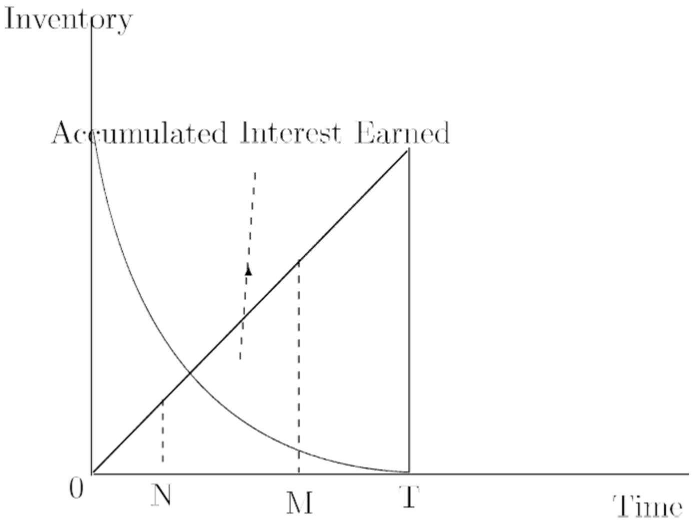

Figure 3: Total amount of interest earned when $M \le T$

description for Scenario I and II, the differential equations representing the inventory status are given by

$$\frac{dI_1(t)}{dt} = R - D, 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} = -D, t_1 \le t \le T \tag{2}$$

with boundary conditions $I_1(0) = 0$ and $I_2(T) = 0$
The solutions of the above differential equations(1) and (2) are given by,

$$I_1(t) = (R - D)t, 0 \le t \le t_1 \tag{3}$$

$$I_2(t) = D(T - t), t_1 \le t \le T \tag{4}$$

put $t = t_1$ (3) and (4), we find the value of t_1 as

$$t_1 = \frac{DT}{(R - D)(1 + D)} \tag{5}$$

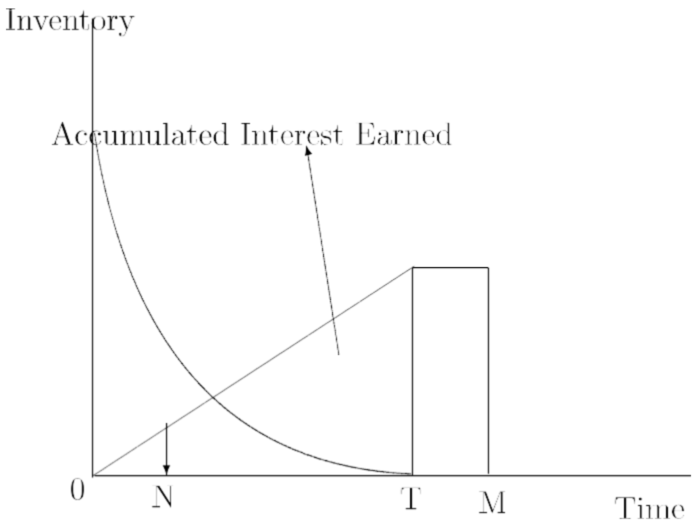

Figure 4: Total amount of interest earned when $N \leq T \leq M$

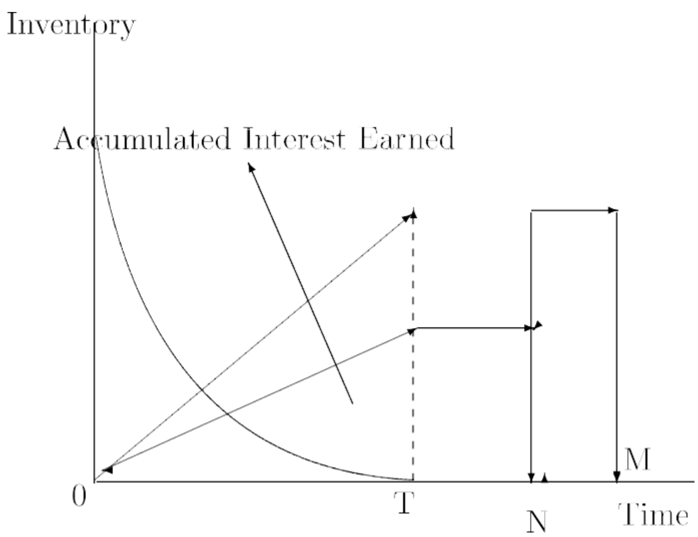

Figure 5: Total amount of interest earned when $T \leq N$

Sincethesupplyrateoccursinthecontinuoustimespan$[0,t_1]$,thentheordersizein the problem is

$$Q = Rt_1 \tag{6}$$

As the defective items are being added in the inventory at the rate of μR^δ per unit time, so the total number of defective items for all scenarios is given by

$$R_1 = \mu R^\delta t_1 \tag{7}$$

Now we want to find the different inventory costs with effect of inflation as:

1. Ordering cost=A

2. The purchase cost

$$PC{=}\int_0^{t_1} PRe^{-rt}dt$$

$$PC = \frac{PR}{r}[1 - e^{-rt_1}] \tag{8}$$

3. Inventory holding cost (HC)for Scenario I and II is given by

$$HC = C_1\left[\int_0^{t_1} I_1(t)e^{-rt}dt + \int_{t_1}^{T} I_2(t)e^{-rt}dt\right]$$

$$HC = C_1\left[(R-D)\left[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\right] + \left[\left(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\right) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\right]\right] \tag{9}$$

4. Refunded cost for Scenario I is given by

$$RFC = C_{rf}\int_0^{t_1} \mu R^\delta e^{-rt}dt$$

$$RFC = \frac{C_{rf}\mu R^\delta}{r}\left[1 - e^{-rt_1}\right] \tag{10}$$

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5.Reworked cost for Scenario II is given by

$$RWC = C_r \int_0^{t_1} \mu R^{\delta} e^{-rt} dt$$

$$RWC = \frac{C_r \mu R^{\delta}}{r}\Big[1 - e^{-rt_1}\Big] \tag{11}$$

The following cases arise due to different types of delay periods

4 Payment Method I

In this case, at the end of the trade credit period(M), the retailer settle the amount for all units sold and keeps the profits for other use and starts paying interest charges on the up-paid balance. To calculate interest payable and interest earned by the retailer,we consider the cases

$$i.\ \ M \leq T \qquad ii.\ \ N \leq T \leq M \qquad iii.\ \ T \leq N$$

Case i $M \leq T$
Annual interest payable

$$= \frac{cI_p}{T} \int_M^T I(t) dt$$

$$= \frac{cI_p}{2}(R - D)\big[T^2 - M^2\big] \tag{12}$$

Case ii $N \leq T \leq M$
Annual interest payable =0
Case iii $T \leq N$
Annual interest payable =0
Similar to interest payable, there are three cases that occur in costs of interest earned per year.
Case i $M \leq T$ Interest earned

$$= sI_e\Big[\alpha \int_0^N Dtdt + \int_N^M Dtdt\Big]$$

$$= sI_e D\Big[\frac{M^2}{2} + \frac{N^2}{2}\big[\alpha - 1\big]\Big] \tag{13}$$

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Caseii $N \leq T \leq M$

Interest earned

$$=sI_e\left[\alpha\int_0^N Dtdt+\int_N^T DNdt+\int_T^M DTdt\right]$$

$$= sI_e\left[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\right] \tag{14}$$

Case iii $T \leq N$

Interest earned

$$= sI_e\left[\alpha\int_0^T DTdt + \alpha\int_T^N DTdt + \int_N^M DTdt\right]$$

$$= sI_e\left[\frac{\alpha T^2}{2} + \alpha TN - \alpha T^2 + DTM - DTN\right] \tag{15}$$

From the above arguments, the annual total cost for the retailer under payment method I can be expressed as

$$TC(T) = \begin{cases} TC_1(T) & \text{if} \quad M \leq T \\ TC_2(T) & \text{if} \quad N \leq T \leq M \\ TC_3(T) & \text{if} \quad T \leq N \end{cases} \tag{16}$$

4.1 Scenario I

Total Cost

Here the total cost per cycle per unit time

$$TC = \frac{1}{T}\left[OrderingCost + PurchaseCost + InventoryHoldingCost + RefundedCost + InterstPayable - InterestEarned\right] \tag{17}$$

Where TC_1=

$$TC_1 = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] + \frac{cI_p}{2}(R-D)\big[T^2-M^2\big] - sI_eD\Bigg[\frac{M^2}{2} + \frac{N^2}{2}[\alpha-1]\Bigg]\Bigg] \quad (18)$$

$$TC_2 = \ldots$$

$$TC_3 = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Bigg[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\Bigg]\Bigg] \quad (19)$$

$$\frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Bigg[\frac{\alpha T^2}{2} + \alpha TN - \alpha T^2 + DTM - DTN\Bigg]\Bigg] \quad (20)$$

4.2 Solution Procedure

$$\frac{dTC_1}{dT} = \frac{-1}{T^2}\Bigg[\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] + \frac{cI_p}{2}(R-D)\big[T^2-M^2\big] - sI_eD\Bigg[\frac{M^2}{2} + \frac{N^2}{2}[\alpha-1]\Bigg]\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} e^{-rT}T - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Bigg) + \frac{cI_p}{2}\Big((R-D)(2T)\Big)\Bigg] \quad (21)$$

Where TC_1=

$$\frac{dTC_2}{dT} = \frac{e^-rt_1 - 1}{T^2} + \frac{PR}{r}[1-e^{-rt_1}] + C_1 \quad - D) \quad + \frac{-}{r} + \quad + \frac{}{r} + \left(\frac{DTe^{-rT}}{-r} \quad \frac{e^{-rT}}{}\right) r^2 \quad -$$

$$r \quad -\frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Big]\Big] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Big[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\Big]$$

$$\frac{dTC_3}{dT} = +\frac{1}{T}\Big[C_1\Big(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Big) - sI_eD\Big[N+M-2T\Big]\Big]$$

$$\frac{e^-rt_1 - 1}{T^2}\Big[\Big[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Big[(R-D)\Big[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Big] + \Big[\Big(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Big) -$$

$$r \quad -\frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Big]\Big] + \frac{C_{rf}\mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Big[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\Big]$$

$$-sI_eD\Big[\alpha T + \alpha N - 2\alpha T + DM - DN\Big] \qquad (23)$$

$$+\frac{1}{T}\Big[C_1\Big(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Big)\Big]$$

i=1,2,3.

By solving the equations $\frac{dTC_i}{dT} = 0$, i=1,2,3, we get the optimal values of $T = T_i$, More over, T satisfies the equations $\frac{d^2TC_i}{dT} > 0$,i=1,2,3. From equations (5)-(7), the optimal value of $t_1 = t_1*$,$Q = Q*$ and $R_1 = R_1*$ can be found out.

4.3 Scenario II

Total Cost

Here the total cost per cycle per unit time

$$TC = \frac{1}{T}\Big[OrderingCost + PurchaseCost + InventoryHoldingCost + ReworkedCost + InterestPayable - InterestEarned\Big] \qquad (24)$$

Where TC_1=

$$TC_{11} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right] + \Bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right] +$$

$$TC_{22} = \frac{cI_p}{2}(R-D)[T^2-M^2] - sI_eD\left[\frac{M^2}{2}+\frac{N^2}{2}[\alpha-1]\right]\Bigg] \quad (25)$$

$$TC_{33} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right] + \Bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right] - sI_e\left[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\right]\Bigg] \quad (26)$$

$$\frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right] + \Bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right] - sI_e\left[\frac{\alpha T^2}{2} + \alpha TN - \alpha T^2 + DTM - DTN\right]\Bigg] \quad (27)$$

4.4 Solution Procedure

$$\frac{dTC_{11}}{dT} = \frac{-1}{T^2}\Bigg[\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right] + \Bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right] + \frac{cI_p}{2}(R-D)[T^2-M^2] - sI_eD\left[\frac{M^2}{2}+\frac{N^2}{2}[\alpha-1]\right]\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_1\left(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}\ldots e^{-rT}T - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\right) + \frac{cI_p}{2}\Big((R-D)(2T)\Big)\Bigg] \quad (28$$

17

Where $TC_1 =$

$$\frac{dTC_{22}}{dT} = \frac{e^{-}rt_1}{-1} \frac{1}{T^2} + \frac{PR}{r}[1-e^{-rt_1}] + C_1 \quad - D) \quad + \frac{}{r} + \quad + \frac{}{r} + \quad r^2 \left(\frac{DTe^{-rT}}{} \quad \frac{e^{-rT}}{} \right)$$

$$r \quad - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r \mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Bigg[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\Bigg]$$

$$\frac{dTC_{33}}{dT} = +\frac{1}{T^2} \frac{1}{T}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Bigg) - sI_eD\Big[N+M-2T\Big]$$

$$\frac{e^{-}rt_1}{-1}\Bigg[\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) -$$

$$r \quad - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r \mu R^{\delta}}{r}\Big[1-e^{-rt_1}\Big] - sI_e\Bigg[\frac{\alpha N^2}{2} + NT - N^2 + MT - T^2\Bigg]$$

$$-sI_eD\Big[\alpha T + \alpha N - 2\alpha T + DM - DN\Big] \tag{30}$$

$$\frac{1}{T}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Bigg)\Bigg]$$

By solving the equations $\frac{dTC_{ii}}{dT} = 0$, i=1,2,3, we get the optimal values of $T = T_{ii}, i = 1, 2, 3$. More over, T satisfies the equations $\frac{d^2TC_{ii}}{dT} > 0, i = 1, 2, 3$.

5 Payment Method II

In this method, a the end of trade credit period(M), if the amount of sales revenue and interest earned is greater than or equal to the purchase cost, the retailer settle the payment owed to the supplier. Otherwise, the retailer pays the supplier the amount of revenue and interest earned and finances the difference. Thereafter, the retailer gradually reduces the financial loan from constant sales and revenue received. To calculate interest payable nd interest earned by the retailer, we consider the cases

$$i.\ M \le T \qquad ii.\ N \le T \le M \qquad iii.\ T \le N$$

Case i Case i $M \le T$

During $[0, M]$ the retailer sells DM units and receives SDM dollars. In addition during thisperiodtheinterestearnedis$\frac{sI_eD}{2}\left(M^2-(1-\alpha)N^2\right)$ Hence,theretailerhas $SDM+\frac{sI_eD}{2}\left(M^2-(1-\alpha)N^2\right)$ dollars at time M.Since the retailer buys DTunits at the purchase cost and the amount, the retailer has at time M, we have the following two sub-cases o calculate the interest charges.

Sub-case i

$$SDM+\frac{sI_eD}{2}\left(M^2-(1-\alpha)N^2\right)\leq cDT$$

Sub-case i

$$SDM+\frac{sI_eD}{2}\left(M^2-(1-\alpha)N^2\right)\geq cDT$$

Let

$$t_w = \frac{sM + \frac{sI_e}{2}\left(M^2-(1-\alpha)N^2\right)}{c}$$

Note that $t_w > M$ for $s \geq c$ then **Sub-case i**-1 $M < t_w \leq T$ In this sub-case, the retailer cannot settle the account at time M. The retailer has to finance the amount $cDT-\left(sDM+\frac{sI_eD}{2}\left(M^2-(1-\alpha)N^2\right)\right)$. The retailer pays off the loan gradually from sales revenue. Hence interest payable per cycle is $\left(\frac{I_p}{2sD}\right)\left(cDT-cDt_w\right)^2$.

Sub-caseii-1$M\leq T\leq t_w$

In this sub-case, the retailer can settle total purchase cost to the supplier at time M. Therefore,there is no interest payable for the retailer.

Case 2 $N \leq T \leq M$

In this sub-case, there is no interest payable.Interest earned is $\left(\frac{sDI_e}{2T}\right)\left(2MT-(1-\alpha)N^2-T^2\right)$

Case 3$T \leq N$

In this case, there is no interest is no interest payable.

Interest earned is $aI_eD(M-(1-\alpha)N-\frac{\alpha T}{2})$, therefore the total cost incurred at the retailer of a supply chain under payment method II

$$TC(T) = \begin{cases} TC_4(T) & \text{if} \quad T \geq t_w \\ TC_5(T) & \text{if} \quad M \leq T \leq t_w \\ TC_6(T) & \text{if} \quad N \leq T \leq M \\ TC_7(T) & \text{if} \quad T \leq N \end{cases} \tag{31}$$

5.1 Scenario1

Total cost

$$TC = \frac{1}{T}\Bigg[OrderingCost + PurchaseCost + InventoryHoldingCost + RefundedCost + InterestPayable - InterestEarned\Bigg] \quad (32)$$

$$TC_{41} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] + \frac{I_p}{2sDT}[cDT - cDt_w]^2 - sI_eD\Bigg[\frac{M^2}{2} + \frac{N^2}{2}[\alpha-1]\Bigg]\Bigg] \quad (33)$$

$$TC_{51} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - \frac{sI_eD}{2}\Bigg[M^2 - (1-\alpha)N^2\Bigg]\Bigg] \quad (34)$$

$$TC_{61} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - \frac{sI_eD}{2}\Bigg(2MT - (1-\alpha)N^2 - T^2\Bigg)\Bigg]$$

$$TC_{71} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-rt_1}}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - sI_eD\Bigg(M - (1-\alpha)N - \frac{\alpha T}{2}\Bigg) \quad (35)$$

5.2 Solution Procedure

$$
\begin{aligned}
\frac{dTC_{41}}{dT} &= \frac{-1}{T^2}\Bigg[\bigg[A+\frac{PR}{r}[1-e^{-rt_1}]+C_1\bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right]+\bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right)- \\
&\quad \frac{e^{-rt_1}}{r}-\frac{e^{-rt_1}}{r^2}+\frac{DTe^{-et_1}}{r}\bigg]\bigg]+\frac{C_{rf}\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right]+ \\
&\quad \frac{I_p}{2sDT}\left[cDT-cDt_w\right]^2-sI_eD\left[\frac{M^2}{2}+\frac{N^2}{2}[\alpha-1]\right]+\frac{1}{T}\bigg[C_1\bigg(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r} \\
&\quad -\frac{e^{-rT}T}{r^2}-\frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\bigg)+\frac{2sDT2D-(cDT-cDt_w)^2(2sD)}{(2sDT)^2}\bigg] \qquad (36)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dTC_{51}}{dT} &= \frac{-1}{T^2}\Bigg[\bigg[A+\frac{PR}{r}[1-e^{-rt_1}]+C_1\bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right]+\bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right)- \\
&\quad \frac{e^{-rt_1}}{r}-\frac{e^{-rt_1}}{r^2}+\frac{DTe^{-et_1}}{r}\bigg]\bigg]+\frac{C_{rf}\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right]-\frac{sI_eD}{2}\left[M^2-(1-\alpha)N^2\right]\bigg]\Bigg] \\
&\quad +\frac{1}{T}\left[C_1\left(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}-\frac{e^{-rT}T}{r^2}-\frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\right)\right] \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dTC_{61}}{dT} &= \frac{-1}{T^2}\Bigg[\bigg[A+\frac{PR}{r}[1-e^{-rt_1}]+C_1\bigg[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right]+\bigg[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right)- \\
&\quad \frac{e^{-rt_1}}{r}-\frac{e^{-rt_1}}{r^2}+\frac{DTe^{-et_1}}{r}\bigg]\bigg]+\frac{C_{rf}\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right]-\frac{sI_eD}{2}\left(2MT-(1-\alpha)N^2-T^2\right)\bigg]\Bigg] \\
&\quad \frac{1}{T}\left[C_1\left(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}-\frac{e^{-rT}T}{r^2}-\frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\right)-\frac{sI_eD}{2}(2T)\right] \qquad (38)
\end{aligned}
$$

$$\frac{dTC_{71}}{dT} = \frac{-1}{T^2} \qquad + \frac{PR}{r}[1-e^{-rt_1}] + C_1 \qquad - D) \qquad + \frac{}{r} + \qquad + \frac{}{r} + \quad r^2 \quad -$$

$$\frac{e^{-}rt_1}{}$$

$$r \quad - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_{rf}\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right] - sI_eD\left(M-(1-\alpha)N-\frac{\alpha T}{2}\right)\Bigg]\Bigg]$$

By solving the equations $\frac{dTC_{ii}}{dT}=0$, i=4,5,6,7 we get the optimal values of $T=T_{ii}, i=4,5,6,7$. Moreover, T satisfies the equations $\frac{d^2TC_{ii}}{dT}>0$, $i=4,5,6,7$

$$\frac{1}{T}\left[C_1\left(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}-\frac{e^{-rT}T}{r^2}-\frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\right)+\frac{sI_eD\alpha}{2}\right] \quad (39$$

5.3 Scenario II

Total Cost

Here the total cost per cycle per unit time

$$TC = \frac{1}{T}\Big[OrderingCost + PurchaseCost + InventoryHoldingCost + ReworkedCost$$

$$+InterestPayable - InterestEarned\Big] \quad (40)$$

Where TC_1=

$$TC_{42} = \frac{1}{T}\left[A+\frac{PR}{r}[1-e^{-rt_1}]+C_1\left[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right]+\left[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right)-\right.\right.\right.$$

$$\frac{e^{-}rt_1}{}$$

$$r \quad - \frac{e^{-rt_1}}{r^2}+\frac{DTe^{-et_1}}{r}\Bigg]\Bigg]+\frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right]+$$

$$\frac{I_p}{2sDT}(cDT-cDt_w)^2 - sI_eD\left[\frac{M^2}{2}+\frac{N^2}{2}[\alpha-1]\right]\Bigg] \quad (41)$$

$$TC_{52} = \frac{1}{T}\left[A+\frac{PR}{r}[1-e^{-rt_1}]+C_1\left[(R-D)\left[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\right]+\left[\left(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\right)-\right.\right.\right.$$

$$\frac{e^{-}rt_1}{}$$

$$r \quad - \frac{e^{-rt_1}}{r^2}+\frac{DTe^{-et_1}}{r}\Bigg]\Bigg]+\frac{C_r\mu R^{\delta}}{r}\left[1-e^{-rt_1}\right]-sI_eD\left[\frac{M^2}{2}-\frac{(1-\alpha)N^2}{2}\right]\Bigg] \quad (42)$$

$$TC_{62} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - \frac{sI_eD}{2}\Bigg(2M-(1-\alpha)N^2-T^2\Bigg)\Bigg]$$

$$TC_{71} = \frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - sI_eD\Bigg(M-(1-\alpha)N-\frac{\alpha T}{2}\Bigg)\Bigg] \quad (43)$$

5.4 Solution Procedure

$$\frac{dTC_{42}}{dT} = \frac{1}{T^2}\Bigg[\frac{1}{T}\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] + \frac{I_p}{2sDT}(cDT-cDt_w)^2 - sI_eD\Bigg[\frac{M^2}{2}+\frac{N^2}{2}[\alpha-1]\Bigg]\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}-\frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\Bigg) + I_p\frac{2sDT2cD-(cDT-cDt_w)^2(2sD)}{(2sDT)^2}\Bigg] \quad (44)$$

$$\frac{dTC_{52}}{dT} = \frac{1}{T^2}\Bigg[\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r}+\frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r}+\frac{e^{-rT}T}{r}+\frac{e^{-rT}}{r^2}\Bigg) - \frac{e^{-}rt_1}{r} - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^{\delta}}{r}\Bigg[1-e^{-rt_1}\Bigg] - sI_eD\Bigg[\frac{M^2}{2}-\frac{(1-\alpha)N^2}{2}\Bigg]\Bigg]\Bigg] + \frac{1}{T}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r}+\frac{De^{-rT}}{r^2}+\frac{e^{-rT}}{r}-\frac{e^{-rT}T}{r^2}-\frac{e^{-rT}}{r^3}+\frac{De^{-rt_1}}{r}\Bigg)\Bigg] \quad (45)$$

$$\frac{dTC_{62}}{dT} = \frac{\frac{e^{-}rt_1}{-1}}{T^2} \quad + \frac{PR}{r}[1-e^{-rt_1}] + C_1 \quad - D) \quad + \frac{}{r} + \quad + \frac{}{r} + \frac{}{r^2} -$$

$$\overline{T} \quad r \quad - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^\delta}{r}\Big[1-e^{-rt_1}\Big] - \frac{sI_eD}{2}\Big(2M-(1-\alpha)N^2-T^2\Big)\Bigg]\Bigg]$$

$$\frac{dTC_{72}}{dT} = \frac{1}{T^2}\Bigg[C_1\Bigg(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Bigg) - \frac{sI_eD}{2}(2T)\Bigg] \quad (46$$

$$\frac{\frac{e^{-}rt_1}{-1}}{\overline{T}}\Bigg[\Bigg[A + \frac{PR}{r}[1-e^{-rt_1}] + C_1\Bigg[(R-D)\Bigg[\frac{e^{-rt_1}}{-r} + \frac{1}{r}\Bigg] + \Bigg[\Bigg(\frac{DTe^{-rT}}{-r} + \frac{e^{-rT}T}{r} + \frac{e^{-rT}}{r^2}\Bigg) -$$

$$r \quad - \frac{e^{-rt_1}}{r^2} + \frac{DTe^{-et_1}}{r}\Bigg]\Bigg] + \frac{C_r\mu R^\delta}{r}\Big[1-e^{-rt_1}\Big] - sI_eD\Big(M-(1-\alpha)N - \frac{\alpha T}{2}\Big)\Bigg]\Bigg]$$

$$1 \Bigg[C_1\Bigg(\frac{De^{-rT}}{-r} + \frac{De^{-rT}}{r^2} + \frac{e^{-rT}}{r} - \frac{e^{-rT}T}{r^2} - \frac{e^{-rT}}{r^3} + \frac{De^{-rt_1}}{r}\Bigg) + \frac{sI_eD\alpha}{2}\Bigg] \quad (47$$

By solving the equations $\frac{dTC_{ii}}{dT} = 0$, i=4,5,6,7 we get the optimal values of $T = T_{ii}, i = 4, 5, 6, 7$. over, T satisfies the equations $\frac{d^2TC_{ii}}{dT} > 0$, $i = 4, 5, 6, 7$.

6 For Non-Defective Items

More

of

$R_1 = 0$. Therefore, the items are completely perfect.Then the optimal results in tis case are obtained from the above expressions using $\mu_\delta = 0$

As the defective items are entered at the rate of μR^δ per unit time, the total number defective items for all scenarios is given by $R_1 = \mu R^\delta t_1$.Note that if $\mu = 0$, then

7 Numerical Example and Sensitivity Analysis

To illustrate the solution procedure and investigate the sensitivity analysis on optimal solution in our model, we consider the following examples.

Example 1 For a retailer who uses payment method I for paying off the loan

Given $A_c = \$50$, a=40,000,b=2.5908,s=\$15,$\eta = 0.4$,p=\$10,$C_r = \$5$, $C_{rf} = \$20$,A=\$100,M=0.853 year,N=0.02 year, $\mu = 0.08$,$\delta = 0.8$,R=500,c=\$1.50,$\alpha = 0.1$

Fordefectiveitems

For Scenario I

We obtain that $t_1 = 1.411$,$T = 0.3542$,We obtain that For Scenario I $Q = 705.5$,$TC_{11} = 26405$,$TC_{21} = 26909$,$TC_{31} = 26910$

$min\{TC_{11}, TC_{21}, TC_{31}\} = min\{26405, 26909, 26910\}$

Therefore the better optimal solution is \$26405

For Scenario II

$Q = 705.5$,$TC_{11} = 26175$,$TC_{21} = 26679$,$TC_{31} = 26679$

$min\{TC_{11}, TC_{21}, TC_{31}\} = min\{26175, 26679, 26679\}$

Therefore the better optimal solution is \$26175

For non-defective items

Scenario I

We use the same parameter values except that of μ and let $\mu = 0$, then we get the optimal solution as:

$Q = 705.5$,$TC_{11} = 25715$,$TC_{21} = 26219$,$TC_{31} = 26220$

$min\{TC_{11}, TC_{21}, TC_{31}\} = min\{25715, 26219, 26220\}$

Therefore the better optimal solution is \$25715

For Scenario II

$Q = 705.5$,$TC_{11} = 25715$,$TC_{21} = 26216$,$TC_{31} = 26680$

$min\{TC_{11}, TC_{21}, TC_{31}\} = min\{25715, 26216, 26680\}$

Therefore the better optimal solution is \$25715

Example 2 For a retailer who uses payment method II for paying off the loan

For Defective

For Scenario I

Using the same parameter values we get the optimal solution $TC_{41} = 15877$,$TC_{51} = 15896$,$TC_{61} = 15890$,$TC_{71} = 15880$

$min\{TC_{41}, TC_{51}, TC_{61}, TC_{71}\} = min\{15877, 15896, 15890, 15880\}$

Therefore the better optimal solution is \$15877

ForScenarioII

Using the same parameter values we get the optimal solution $TC_{42} = 15606, TC_{52} = 15610, TC_{62} = 15615, TC_{72} = 15620$

$min\{TC_{42}, TC_{52}, TC_{62}, TC_{72}\} = min\{15606, 15610, 15615, 15620\}$

Therefore the better optimal solution is \$15606

For Non-Defective

For Scenario I

Using the same parameter values we get the optimal solution $TC_{41} = 15471, TC_{51} = 15475, TC_{61} = 15479, TC_{71} = 15485$

$min\{TC_{41}, TC_{51}, TC_{61}, TC_{71}\} = min\{15471, 15475, 15479, 15485\}$

Therefore the better optimal solution is \$15471

For Scenario II

Using the same parameter values we get the optimal solution $TC_{42} = 15606, TC_{52} = 15610, TC_{62} = 15615, TC_{72} = 15620$

$min\{TC_{42}, TC_{52}, TC_{62}, TC_{72}\} = min\{15606, 15610, 15615, 15620\}$

Therefore the better optimal solution is \$15606

7.1 Sensitivity Analysis

Let us consider the same data as in example. Here, we study the effects of changes in the values, on optimal cycle and minimum total cost.

Figure 6: Payment Method I For Defective Items

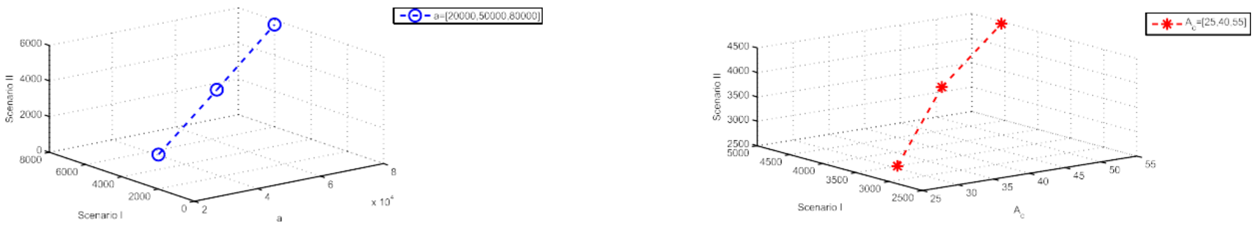


Table 1: Payment Method II For Defective Items

A_c	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
25	0.02	0.8099	1.8680	2879.2	2872.9	404.9500
40	0.06	1.1412	1.9945	3832.5	3807.7	570.6000
55	0.10	1.4502	2.1552	4554.7	4506.2	725.1000
a	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
20000	0.14	0.5225	1.8266	1988.9	1960.0	261.2500
50000	0.18	1.1412	1.9945	3981.1	3906.8	570.6
80000	0.22	2.3700	2.7615	6012.7	5876.5	1185
b	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2.4	0.26	3.4034	3.5482	6793.3	6613.4	1701.7
2.8	0.30	1.1412	1.9945	4129.6	4005.8	570.6000
3.0	0.34	0.5934	1.8277	2392.3	2312.7	296.7
η	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.3	0.38	0.4789	1.8303	1958.5	1886.8	239.45
0.5	0.42	1.1412	1.9945	4278.2	4104.9	570.600
0.7	0.46	16.1307	12.8929	9409.7	8994.5	8065.4
C_{rf}	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
10	0.46	1.1385	1.9899	4137.8	4123.8	569.25
20	0.44	1.1412	1.9945	4484.6	4121.4	570.6000
30	0.41	1.1126	1.9991	4644.0	3985.6	556.3
A	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
50	0.41	1.1438	1.9446	4360.1	4186.1	571.900
150	0.39	1.1412	1.9945	4266.2	4105.2	570.6000
200	0.37	1.1692	2.0435	4263.7	4111.0	584.6
C_1	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2	0.37	1.12140	2.1219	3634.4	3493.3	607.0000
4	0.35	1.1412	1.9945	4473.9	4329.5	570.6000
6	0.32	1.0794	1.8866	5005.1	4873.1	539.7
r	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.05	0.30	1.0786	1.8853	4115.2	3991.4	539.3
0.09	0.27	1.1412	1.9945	4087.4	3975.9	570.6000
0.13	0.21	15.0948	26.3826	4021.1	3916.2	7547.4

Table2: Payment Method II For Defective Items

A_c	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
25	0.8099	1.8680	2876.2	2863.5	404.9500
40	1.1412	1.9945	3829.0	3779.5	570.6000
55	1.4502	2.1552	4554.7	4457.7	725.1000
a	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
20000	0.5225	1.8266	1987.1	1931.1	261.2500
50000	1.1412	1.9945	3979.4	3832.5	570.6
80000	2.3700	2.7615	6009.5	5740.3	1185
b	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2.4	3.4034	3.5482	6793.3	6433.5	1701.7
2.8	1.1412	1.9945	4129.6	3882.0	570.6000
3.0	0.5934	1.8277	2392.3	2233.0	296.7
η	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.3	0.4789	1.8303	1958.5	1815.1	239.45
0.5	1.1412	1.9945	4278.2	3931.5	570.600
0.7	16.1307	12.8929	9409.7	8579.4	8065.4
C_{rf}	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
10	1.1385	1.9899	4137.8	3947.9	569.25
20	1.1412	1.9945	4484.6	3939.8	570.6000
30	1.1126	1.9991	4644.0	3821.0	556.3
A	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
50	1.1438	1.9446	4360.1	4012.1	571.900
150	1.1412	1.9945	4266.2	3944.2	570.6000
200	1.1692	2.0435	4263.7	3958.3	584.6
C_1	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2	1.12140	2.1219	3633.3	3351.2	607.0000
4	1.1412	1.9945	4473.9	4185.0	570.6000
6	1.0794	1.8866	5005.1	4741.0	539.7
r	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.05	1.0786	1.8853	4115.2	3867.6	539.3
0.09	1.1412	1.9945	4087.4	3864.5	570.6000
0.13	15.0948	26.3826	4002.9	3829.5	7547.4

Table3: Payment Method I For Non-Defective Items

A_c	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
25	0.02	0.8099	1.8680	2860.4	2857.2	404.9500
40	0.06	1.1412	1.9945	3840.5	3754.2	570.6000
55	0.10	1.4502	2.1552	4405.5	4494.0	725.1000
a	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
20000	0.14	0.5225	1.8266	1899.1	1960.0	261.2500
50000	0.18	1.1412	1.9945	3840.5	3754.7	570.6
80000	0.22	2.3700	2.7615	5694.7	5599.7	1185
b	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2.4	0.26	3.4034	3.5482	6347.5	6248.6	1701.7
2.8	0.30	1.1412	1.9945	3840.5	3754.7	570.6000
3.0	0.34	0.5934	1.8277	2232.4	2150.3	296.7
η	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.3	0.38	0.4789	1.8303	1822.4	1740.2	239.45
0.5	0.42	1.1412	1.9945	3840.5	3754.7	570.600
0.7	0.46	16.1307	12.8929	8262.4	8157.8	8065.4
C_{rf}	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
10	0.46	1.1385	1.9899	3840.4	3754.7	569.25
20	0.44	1.1412	1.9945	3840.5	3754.7	570.6000
30	0.41	1.1126	1.9991	3738.8	3652.9	556.3
A	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
50	0.41	1.1438	1.9446	3919.6	3834.8	571.900
150	0.39	1.1412	1.9945	3962.5	3877.8	570.6000
200	0.37	1.1692	2.0435	3888.8	3802.1	584.6
C_1	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2	0.37	1.12140	2.1219	3557.8	3469.8	607.0000
4	0.35	1.1412	1.9945	4121.8	4036.0	570.6000
6	0.32	1.0794	1.8866	4685.9	4602.4	539.7
r	μ	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.05	0.30	1.0786	1.8853	3827.3	3743.8	539.3
0.09	0.27	1.1412	1.9945	3838.9	3753.1	570.6000
0.13	0.21	15.0948	26.3826	124.7606	19.8961	7547.4

Table 4: Payment Method II For Non-Defective Items

A_c	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
25	0.8099	1.8680	1547.1	1550.5	404.9500
40	1.1412	1.9945	3829.0	3779.5	570.6000
55	1.4502	2.1552	4405.4	4400.8	725.1000
a	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
20000	0.5225	1.8266	1899.1	1897.2	261.2500
50000	1.1412	1.9945	3754.7	3751.1	570.6
80000	2.3700	2.7615	5599.7	5594.5	1185
b	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2.4	3.4034	3.5482	6248.6	6245.6	1701.7
2.8	1.1412	1.9945	3754.7	3751.3	570.6000
3.0	0.5934	1.8277	2150.3	2147.9	296.7
η	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.3	0.4789	1.8303	1740.2	1736.0	239.45
0.5	1.1412	1.9945	4278.2	3931.5	570.600
0.7	16.1307	12.8929	8157.8	8155.0	8065.4
C_{rf}	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
10	1.1385	1.9899	3754.7	3752.0	569.25
20	1.1412	1.9945	3738.8	3652.9	570.6000
30	1.1126	1.9991	3877.8	3875.4	556.3
A	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
50	1.1438	1.9446	3877.8	3874.1	571.900
150	1.1412	1.9945	3802.1	3800.6	570.6000
200	1.1692	2.0435	3469.8	3466.6	584.6
C_1	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
2	1.12140	2.1219	4036.0	4033.1	607.0000
4	1.1412	1.9945	4602.4	4599.9	570.6000
6	1.0794	1.8866	3743.8	3722.2	539.7
r	t_1	T	$Scenario I Total Cost$	$Scenario II Total cost$	Q
0.05	1.0786	1.8853	3743.8	3738.8	539.3
0.09	1.1412	1.9945	3753.1	3749.3	570.6000
0.13	15.0948	26.3826	3742.8	3739.1	7547.4

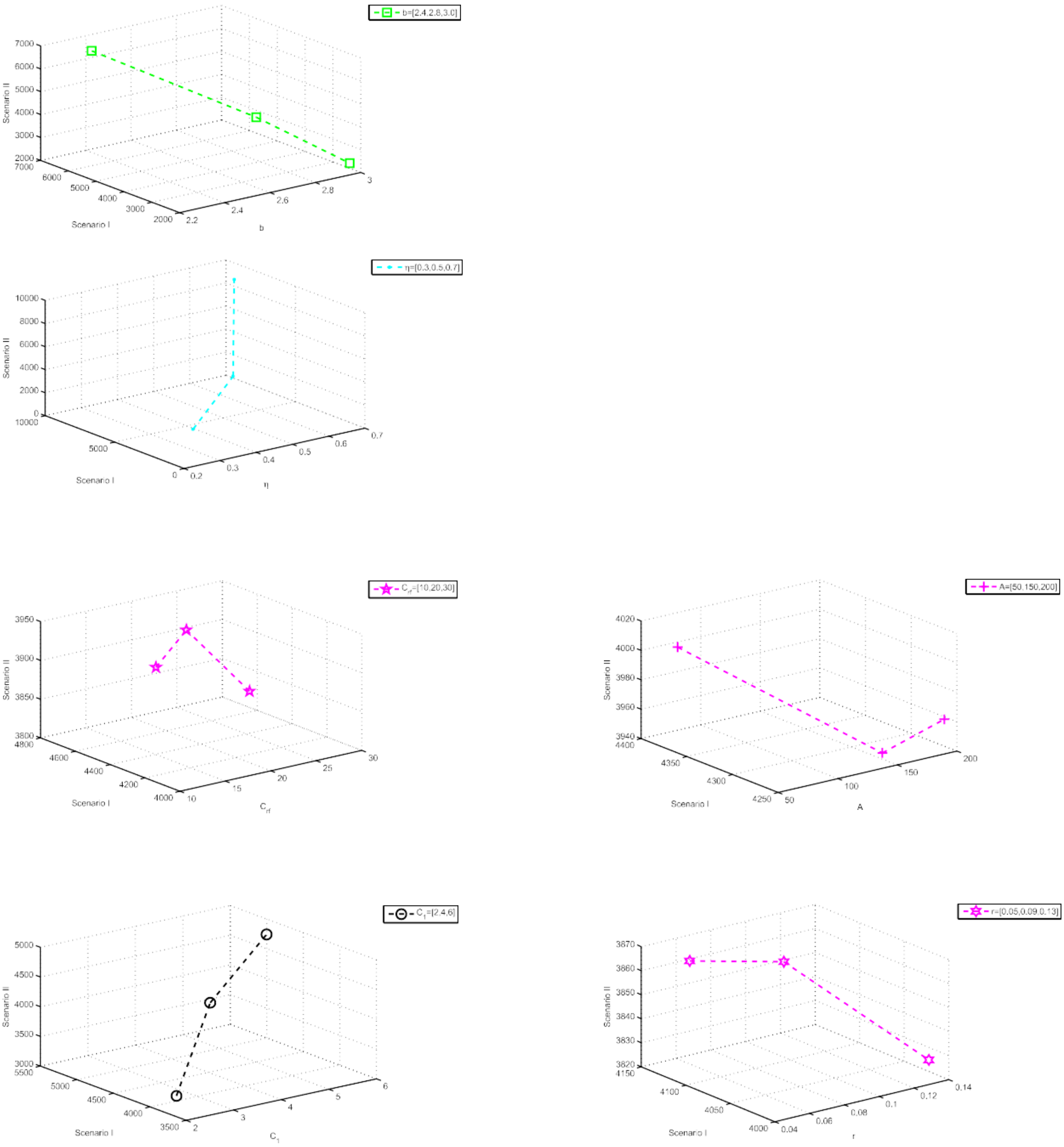
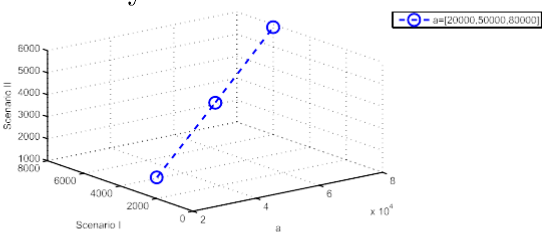
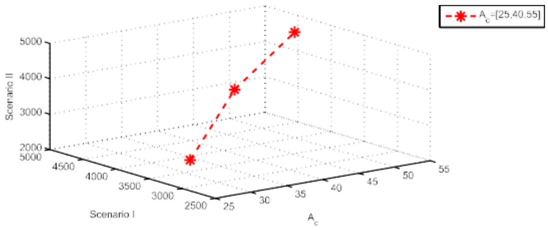


Figure 7: Payment Method II For Defective Items





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2000
Scenario I
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4500
4000
3500
3000
2500
A_c
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35
40
45
50
55

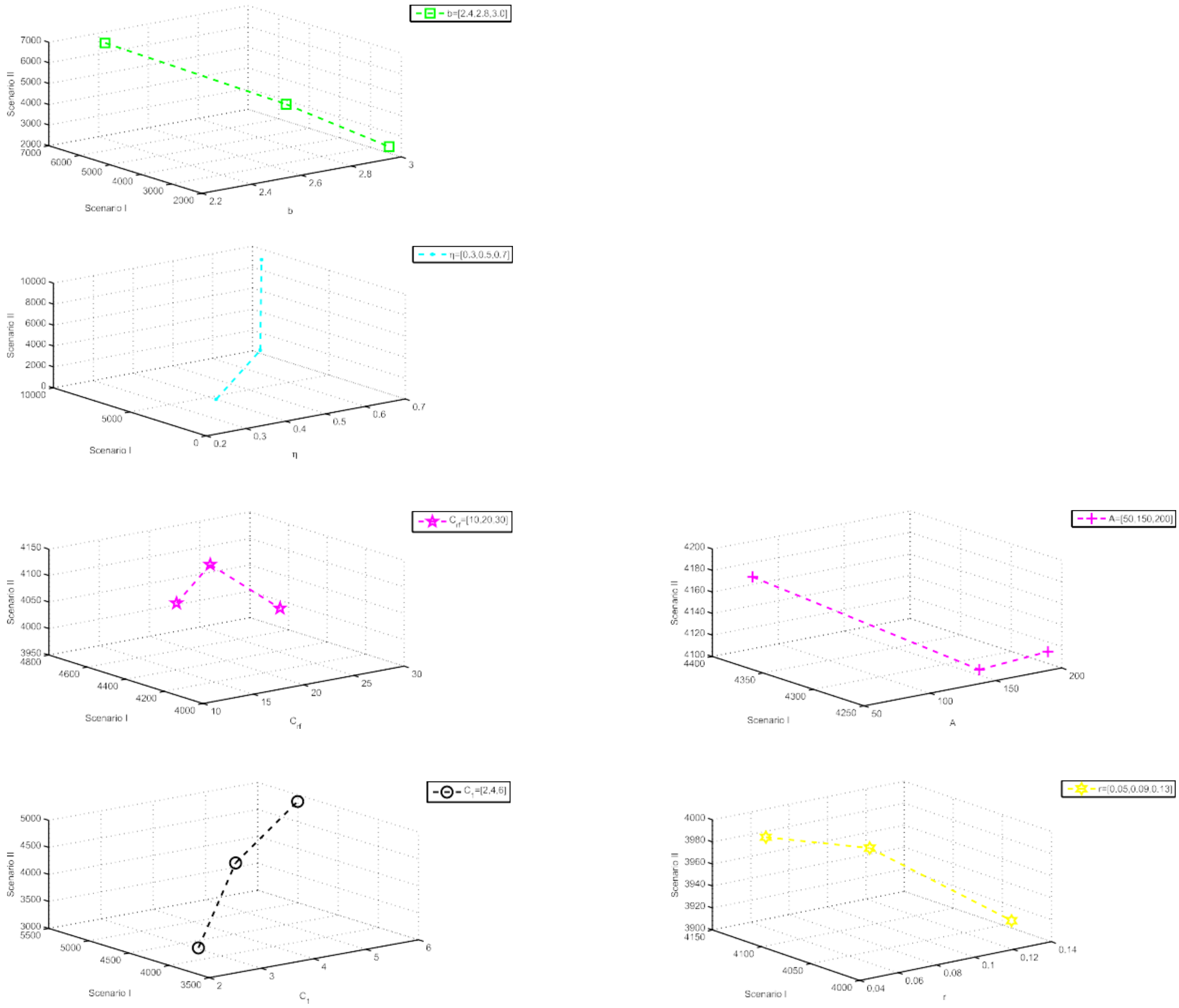


Figure 8: Payment Method I For Non- Defective Items

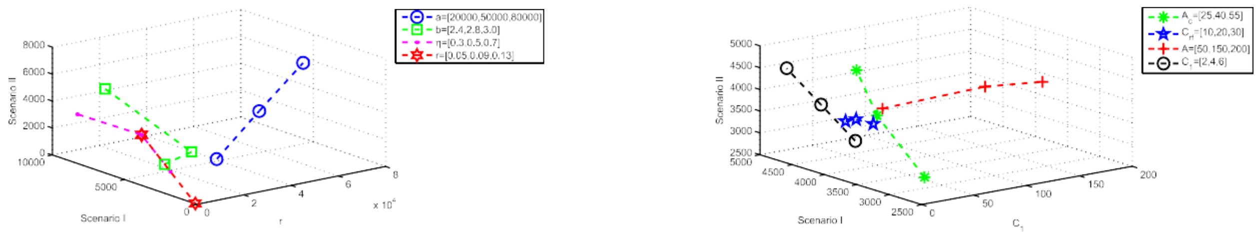
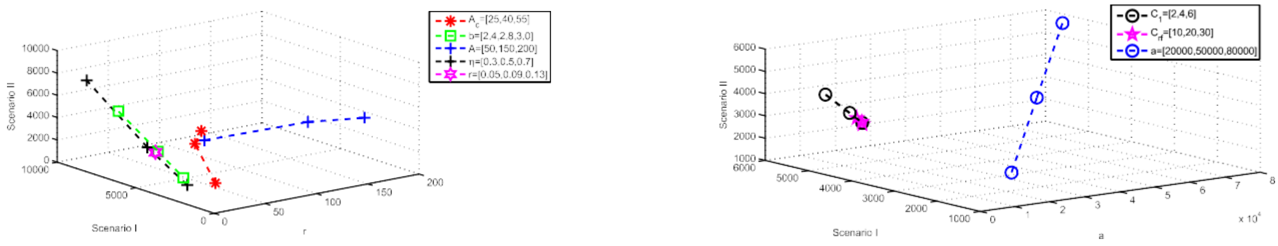


Figure 9: Payment Method II For Non- Defective Items



8 ManagerialImplication

For defective item in both Payments Table 1 and Table 2

1. When A_c value increasing, the total cost of Scenario I and II also the order quantity are highly increasing.

2. When parameter a increasing, the total cost of Scenario I and II also the order quantity are highly increasing.

3. When parameter b increasing, the total cost of Scenario I and II also the order quantity are highly decreasing.

4. When parameter η increasing, the total cost of Scenario I and II also the order quantity are highly increasing.

5. When refunded cost C_{rf} increasing, the total cost of Scenario I and II also the order quantity are changes variably.

6. When setup cost A increasing, the total cost of Scenario I and II also the order quantity are changes variably.

7. When holding cost C_1 increasing, the total cost of Scenario I and II are highly increasing.

8. When parameter r increasing, the total cost of Scenario I and II also the order quantity are decreasing.

For non- defective item in both Payments Table 3 and Table 4

1. When A_c value increasing, the total cost of Scenario I and II also the order quantity are highly increasing.

2.Whenparameteraincreasing,thetotalcostofScenarioIandIIalsotheorder quantity are highly increasing.

3. When parameter b increasing, the total cost of Scenario I and II also the order quantity are highly decreasing.

4. When parameter η increasing, the total cost of Scenario I and II also the order quantity are highly increasing.

5. When refunded cost C_{rf} increasing, the total cost of Scenario I and II also the order quantity are changes variably.

6. When setup cost A increasing, the total cost of Scenario I and II also the order quantity are changes variably.

7. When holding cost C_1 increasing, the total cost of Scenario I and II are highly increasing.

8. When parameter r increasing, the total cost of Scenario I and II also the order quantity are decreasing.

9 Conclusion

In this paper, we first implemented to different payment methods for the retailer to pay off the loan to the supplier under two echelon trade credit scenario. In this present situation, inflation and time value of money are also the main factors. In keeping with this reality, these factors are incorporated in present work. Finite replenishment rate, price and advertisement dependent deterministic demand pattern are considered in this model and two different scenarios are discussed. Based on this scenarios, the imperfect items are reworked or if they reach the customer, refunded.Numerical examples are given to illustrate the model. Sensitivity analysis for the effects of the parameters on the decisions are also offered. To archive optimized trade credit policies, which is helpful for the supply chain, the supplier should share additional profits to encourage the retailer to cooperate.

In future research, our model can be extended in several ways. We could extend the model by considering the non-zero lead time. Also, we may consider time dependent holding cost. Finally we could extend this model by allowing shortages.

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