

## LIRP Model: Distribution of World Food Program Multi-Modal Transportation in Congo

Asthy Jover Nathanael Massengo Milandou

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### ABSTRACT

The location Inventory-routing problem's objective here in this case of WFP in Congo is to select facility locations, assign Delivering point to facilities, and design routines between warehouses and Delivering point. The main aims are to achieve minimal cost. Generally for the standard location Inventory-routing problem, the fixed costs to open Distribution Centers (DCs) is included in total cost plus the transportation costs from facilities to Delivering point (schools). This paper also considers the facilities operations variable cost. There is two proposed variable cost forms. One is a linear cost function with a constant variable (operating) cost per unit; the other employs a concave function of total throughput at any DC as its cost model. The latter is studied because economies case can be achieved for Wide facilities. By economies case, we mean that the variable cost per unit is a decreasing function of the number of units of throughput, that link is expressed through a concave function.

Two solution methods are developed, both based on a genetic algorithm. After some preliminary tests, one approach is employed for further testing. Computational experiments of the model without variable cost are performed on published data sets. Then extensive testing is done on modified data sets for cases with operating cost but without economies of scale, and for other cases when economies of scale are present. Analysis of the influence of economies of scale is provided. First, we briefly test how parameter values affect the economies of scale. Then we extensively analyze the tradeoffs between operating costs of facilities and transportation costs. Conclusions are drawn and further research is suggested.

**KEYWORDS:** DISTRIBUTION; LIRP; WORLD FOOD PROGRAM

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### I. INTRODUCTION

The Location Inventory routing problem (LIRP) is to find the minimum cost route to be traveled by a fleet or vehicles. The container loading problem's (CLP) role is mainly used to design the loading solution with high quality, in order to increase the utilization rate of vehicles, which can reduce the number of employed vehicles charge and transportation cost.

Location search is also one of the tough part in WFP distribution, when it comes to find the location of the disaster area or school to deliver (site).

Depot unload problem is when we realized that the depot at the site has not the required conditions of a warehouse and labor of unloading and handling good from the vehicle.

All these kind of situations have been widely revised but the combination research of them wasn't planned to start the activities, everything was made and improvised according to the life reality in regions. Taking about WFP in Congo Brazzaville, Africa.

The distribution method that WFP used is fixing-time and fixing-route for distribution to the daily actual dynamic demand from sites in regions. And when we are in low season or the peak season, the WFP employs less amount of vehicle which will indeed not increase the no-load ratio and the transportation costs.

How to solve the distribution problem of school delivery in the shortest distance and improve the utilization rate of vehicles. All these problem should be addressed immediately to the distribution department.

The character of CVRP in WFP is that each vehicle must meet the loading capacity constraints which usually include the restriction of total weight or total volume, while in real life of WFP, the loading capacity are often more complex.

The Location inventory routing problem has been extensively surveyed since it has been formally first introduced by Dantzig and Ramser in 1959 and a lot of companies branches are derived from this problem. The LIRP is one of the most common and the most important research branches. The character of LIRP is that every single vehicle is supposed to meet the maximal loading capacity constraints which usually include restriction of total weight or total volume, while in WFP, the loading capacity constraint are often more complex, like during

the Transport and distribution, if you can't find a suitable loading program to put all the required items of delivery points demand into the same vehicle then we cannot use vehicle to achieve all the distribution of listed site (schools), so we get to the point that we must use more vehicle or we must put those goods into different vehicles in order to distribute all the goods. This will surely increase the transportation cost and then we attached multi-dimensional loading constraint to the CVRP

## **II. LITERATURE REVIEW**

In this 21st century time, the human information technology became rapid; easy and developed. It is also a period where we find a surge of public social problems such as population, environment and resources that gradually increased in the number of public and natural disasters. The global economic losses are more than \$3 trillion. Around 1997 and 2015, all sort of natural disasters have impacted a lot of people in Africa and let's talk about (Congo) resulting in an economics. Either natural damages or human accidents, a significant loss of personal and property can easily be caused by emergencies if the rescues are not achieved effectively, the consequences of these secondary disasters will be even more serious. As the most important part of the emergency management system, emergency logistics has seriously got attention from everyone. In order to make in emergency management and rescue processes more efficient, it is critical to select the emergency supply locations and the routes between the warehouses and the affected areas. Therefore, the study of Location Inventory-Routing Problem (LIRP) emerges. The goal of this paper is to study single-stage LIRP to provide theoretical support for the practical work of emergency management and emergency logistics.

The survey of logistics in emergency is relatively early, and most scholars have studied from the aspects of emergency logistics management, routing, site selection of emergency supplies storage, etc., by establishing mathematical models, and gradually form a complete systematic research topic. For the first time, Kembalcook and Stephenson proposed the application of logistics management for emergency activities in the refugee relief operation in Somalia and developed that the logistics should first be centralized into a single organization in the event of a major emergency, which improves the transportation efficiency of emergency supplies. Better understanding the unexpected events and incident management operations, Tufekci and Wallace not only emphasized the importance of policies, but also applied advanced communication and computer technology to build an emergency logistics mathematical model. F Fiedrich et al. developed a dynamic optimization model for how to find available resources and how to distribute them after strong earthquakes and presented detailed descriptions of the available resources and operational areas. Altay and Green summarized the previous literatures to find out the potential research directions on the issue of disaster operations and provided a starting point to the researchers. R Oloruntoba et al. applied the idea of customer service in international emergency relief chain management and verified that the customer service played an important role in the operation strategies. Nahleh et al. made predisaster plans using an integrated forecasting method and established a predictive tool with regard to the best fit probability distribution of four variables: number of disasters, number of deaths, number of people affected, and economic loss. In the paper of Liu and Ye, a multistage humanitarian logistics planning model based on Bayesian Group Information Update (GIU) was stated. This approach was verified that it can allocate emergency supplies well according to the latest information. Later, Y Ye et al. developed their previous study and discussed the loss of resources allocation and the loss of emergency logistics time. A two-stage scheduling method considering stochastic demand and travel time was proposed. Meantime, a deployment approach based on Bayesian GIU was proposed considering the incompleteness and lack of information of the actual situations.

A multi-criteria analysis method was developed through conflicting multi-objectives in the hierarchy. The top decision makers worked together to design an interactive program to evaluate the preferences of alternative non-dominated solutions. L. Özdamar et al. studied the multi-commodity network flow and vehicle routing in order to integrate the process of transporting materials into decision support system and formulated the logistics planning model under natural disasters on the basis of above study that solved by Lagrangian relaxation. G. Barbaroso et al. proposed a two-stage stochastic programming model for the complexity of emergency rescue supplies to the disaster area after analyzing the system uncertainty and information asymmetry caused by the vulnerability of the transportation system. B. Balcik et al. studied the location selection decision (Inventory) of the humanitarian relief chain in response to sudden disasters, made a variant model based on the largest coverage location model, and discussed its management significance. A. M. Caunhye et al. developed a location configuration model to locate alternative medical facilities and designed a stochastic assignment scheme in a catastrophic radiation event primarily considering the factor of the choice of therapeutic methods and automatic evacuation. Y. Rahmani et al. addressed mixed integer linear model for the secondary location and routing selection including multiproduct and transportation and used the Complex solver to solve this small-scale problem. Jiahong Zhao and Ke studied the environmental risks of explosive waste management from the aspect of facility location, inventory level, multi-warehouse vehicle past, etc... In Jiahong Zhao and

Ginger Y. Ke study, they formulated a solution based on TOPSIS method with reasonable calculation time to test their approach's effectiveness.

Wherever the overhead researches on emergency logistics and emergency LIRP, we can find that (1) most scholars divide LIRP into a two-stage issue, which is site selection and route planning, and use them to the different cases without making full use of the relation of LAP and VRP. (2) The utilization of classical algorithms is too trendy, and classical algorithms have many shortcomings. A lot of new algorithms are more effective than classical algorithms. (3) Most of studies focus on one case; however, they don't formulate a method model that can be used in multiple context settings.

According to the real humanitarian life, we consider that the location of the warehouse and route of the delivery vehicle have strong dependence in the emergency logistics system. So, a single-stage LIRP model with the minimum rescue time and the lowest cost is established.

LIRP is a typical NP-hard problem, which needs an optimization algorithm inspired by nature. In 2010, Yang developed the bat algorithm (BA) that mimics the microbats behaviors of echolocation for orientation and prey seeking. It has received much attention, since the BA is simpler and has fewer parameters than other swarm intelligence algorithms. Anyways, like other communal intelligence, the original BA is also easy to fall into local extremum and it is slow convergence at a later stage. Therefore, many of the effort in the bat algorithm research focuses on the improvement of the BA performance.

He et al. introduced both simulated annealing and Gaussian perturbations into the original BA and called the new method as the simulated annealing Gaussian bat algorithm (SAGBA). This new algorithm inherits the simplicity and efficiency of the original BA and speeds up the global convergence rate for the global optimality. Anyways, since the search iterations continue, the temperature is reduced, so the SAGBA maintains the standard BA's characteristics.

Gandomi and Yang presented a chaotic bat algorithm (CBA) and applied deterministic chaotic signals in place of constant values. The results suggest that the new algorithm can improve the reliability of global optimality. Jordehi also proposed a chaotic-based bat algorithm, which can diversify the bats and mitigate premature convergence problem by the ergodicity and non-repetitious nature of chaotic functions. Jun et al. introduced a double subgroup with a dynamic transition strategy into the standard BA to improve global exploring ability. Ramli et al. improved the phenomenon of slow convergence rate and low accuracy by modifying the dimensional size and providing inertia weight. From simulations, the new algorithm proves to be more effective than the standard BA in terms of searching for a solution. Hong et al. forecasted the motion of a floating platform by a support vector regression model with a hybrid kernel function and proposed chaotic efficient bat algorithm based on the chaotic, niche search, and evolution mechanisms to improve the reliability and effectiveness of the basic BA.

Fister et al. developed self-adaptation bat algorithm (SABA) from differential evolution and tested on ten benchmark functions from publications. Fister Jr. et al. proposed the Hybrid Bat Algorithm (HBA) to improve the original bat algorithm by developing new variant with differential evolution (DE) strategy. The DE strategy maintains a population of candidate solutions and creates new candidate solutions that have the best score or fitness to optimize the problem. So, the cross-generation will not be easy to get rid of the local optima if ever these two solutions are in the same local optima region.

Fister dissertation, a novel Hybrid Self-Adaptive Bat Algorithm (HSABA) is proposed, which enables a self-adaptation of its control parameters and the DE strategy. The results proved that the HSABA outperforms the results of the basic BA and the SABA.

In this paper, we use HSABA to solve the single-stage LRP problem. The HSABA not only uses the self-adaptation mechanism.

### **III. APPLIED MODEL**

#### **3.1 Problem Definition:**

We consider the location Inventory-routing problem with capacity on both depots and vehicles and introduce variable cost of distribution centers to the formulation. The optimal locations of distribution centers are chosen from a set of practical potential locations, considered in account that vehicle routes must be operated from the chosen site. That is, School of delivery should be assigned to the opened distribution center and routes need to be constructed.

For the model without economies case, the previous variable cost of a distribution center is expressed as a unit operating cost multiplied by the total demand. When economies case are present, a concave function of facility throughput is employed for calculating the variable cost of a depot. All both formulations have the objective of achieving minimal total cost.

Many assumptions pertain to our model:

- The vehicles are homogeneous, each with the same capacity.
- There is several products type.

- Facilities can have deferent capacities, but are the same in all other aspects. In other words, each facility has the same unit operating cost.

3.2 Notation:

The notation below applies to all both of our proposed models, i.e. with and without the Economies case. The location inventory routing problem with economies of scale can be characterized by the following parameters:

- $I$ : set of indices of possible locations of DCs
- $J$ : set of indices of customers
- $L: L= I \cup J$ : set of nodes
- $K$ : set of indices of customers
- $A_i$ : fixedcode cost of DC  $i, \forall i \in I$
- $U$ : unit operating cost of DCs for the case without economies of scale
- $C_{ji}$ : transportation cost from node  $i$  to node  $j, \forall i, j \in L$
- $W$ : vehicle capacity
- $F$ : fixed cost per vehicle
- $Q_i$ : maximum throughput (capacity) of DC  $i, \forall i \in I$
- $d_j$ : demand of customer  $j, \forall j \in J$

Variable

**Variable**

1if note  $i$  is immediate predecessor of note  $j$  on the vehicle  $k \forall i, j \in L, \forall k \in L$

$$- x_{ijk} = \begin{cases} 1 & \text{if note } i \text{ is immediate predecessor of note } j \text{ on the vehicle } k \\ 0 & \text{other wise} \end{cases}$$

$$- v_{ij} = \begin{cases} 1 & \text{if customer } j \text{ is reserved by DC } i \\ 0 & \text{Otherwise} \end{cases}$$

$$- y_i = \begin{cases} 1 & \text{if DC } i \text{ is open} \\ 0 & \text{otherwise} \end{cases}$$

The model objective is to select locations, at the same time, find best routes with the lower total cost, it includes the fixed cost to locate a facility, nonlinear

Operating cost, and transportation cost. In addition, the following constraints should be satisfied:

- Each Delivery School must be assigned to a single route
- Each route starts and ends at a DC
- The total load of every vehicle is within its capacity limit.
- The total quantity of sites served by any DC cannot exceed the capacity of that DC.

The problem can be designed according on the classical location-routing model and a location problem with economies case. The formulation presented below begins with the location routing model given by Prins et al. (2006a). Our model is different from that of Prins et al. by the addition of the variable cost term in the objective function, Anyways, only in small change in notation. The constraints (3.2) through (3.11) in Sec. 5.3 are identical to those of Prins et al. (2006a). The objective function in the model [LRPV] adds a variable-cost term,  $\sum_{j \in J} U d_j$ , which is not considered by Prins et al.

Note that when economies of scale are absent, the operating cost (variable cost) for that model simply represents a constant addition to the objective function of models of Prins et al. (2006a) and others that ignore operating cost. We involved in it the variable-cost term so that we may compare the results without economies

caseto those when economies case are present. The latter model, [LRPES], is presented in Sec. 5.4. The constraints are unchanged; the variable-cost term is replaced by Eq. (3.1) below.

That variable-cost term in the objective function of the model [LRPES] thus ensures that the operating cost exhibits concavity. The concave variable cost function is  $f_i$ ; its argument is the total throughput of facility  $i$ :

$$f_i = \sum_{j \in J} d_j v_{ij} = \gamma \left( \sum_{j \in J} d_j v_{ij} \right)^\delta$$

That total throughput is summed over each facility  $i$  in [LRPES]. The non-linear cost function is illustrated in **Figure 3.1**, in which we set  $\gamma=25$  and  $\delta=2/3$ . With an increase in the quantity handled at a depot, the operating cost per unit (the slope of the curve) decreases.

Attainment of such economies case requires that  $0 < \delta < 1$  in Eq. (3.1).

### 3.3 Pattern Survey without Economies:

The location Inventory-routing problem with variable cost but no economies case is

$$[LRPV] \min z = \sum_{i \in I} A_i y_i + \sum_{j \in J} U d_j + \sum_{i \in L} \sum_{j \in L} \sum_{k \in K} C_{ij} x_{ijk} + \sum_{i \in L} \sum_{j \in J} \sum_{k \in K} F x_{ijk}$$

Subject to

$$(3.2)$$

$$\sum_{j \in J} \sum_{i \in L} d_j x_{ijk} \leq W \quad \forall k \in K \quad (3.2)$$

$$\sum_{j \in J} \sum_{i \in L} d_j x_{jik} \leq W \quad \forall k \in K \quad (3.3)$$

$$\sum_{j \in J} x_{ijk} - \sum_{j \in L} x_{jik} = 0 \quad \forall k \in K, \forall i \in L \quad (3.4)$$

$$\sum_{i \in L} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (3.5)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \quad \forall S \in J, \forall k \in K \quad (3.6)$$

$$\sum_{u \in J} x_{iuk} + \sum_{u \in L/\{j\}} x_{ujk} \leq 1 + v_{ij} \quad \forall i \in I, \forall j \in J, \forall k \in K \quad (3.7)$$

$$\sum_{j \in J} d_j v_{ij} \leq Q_i y_i \quad \forall i \in I \quad (3.8)$$

$$y_i \in \{0,1\} \quad \forall i \in I \quad (3.9)$$

$$v_{ij} \in \{0,1\} \quad \forall i \in I, \forall j \in J \quad (3.10)$$

$$x_{ijk} \in \{0,1\} \quad \forall i \in L, \forall j \in L, \forall k \in K \quad (3.11)$$

In The constraints will be discussed the following section.

### 3.4 Pattern Survey with Economies:

$$[LRPES] \min z = \sum_{i \in I} A_i y_i + \sum_{i \in I} f_i \left( \sum_{j \in J} d_j v_{ij} \right) + \sum_{i \in L} \sum_{j \in L} \sum_{k \in K} C_{ij} x_{ijk} + \sum_{i \in L} \sum_{j \in J} \sum_{k \in K} F x_{ijk}$$

subject to

$$(3.2)-(3.11)$$

We count two models similar here; differences lie in the objective functions. The objective function of each model is the sum of the operating cost including fixed cost, variable cost of operation, and transportation cost between depots and sites. In Section 3.3, the variable cost is calculated by multiplying the total output by the unit operating cost. Since that unit cost is a constant, the total operating cost in [LRPV] can be calculated directly by the total needs of all sites.

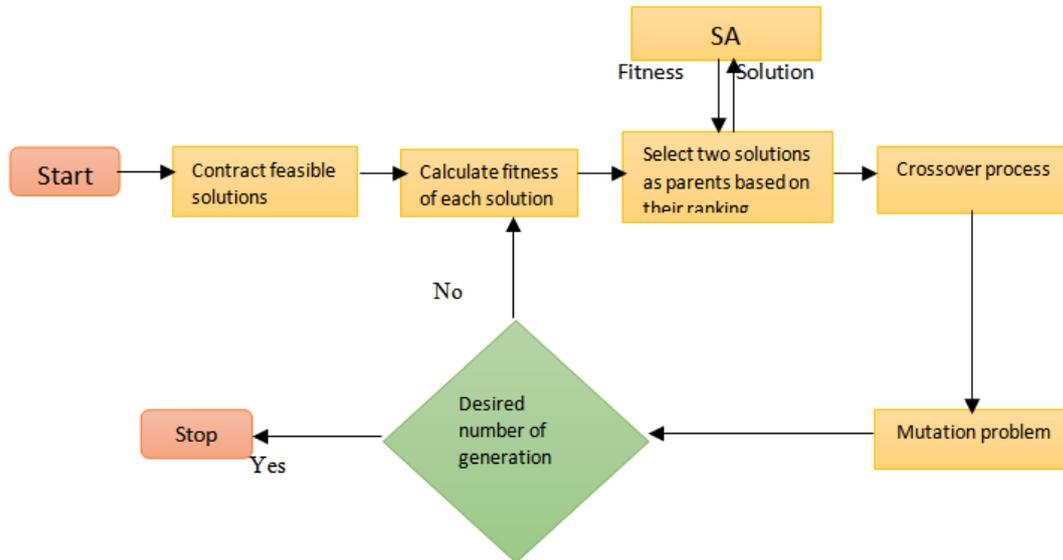
*Function* of Eq. (3.1); In [LRPES], the peripheral operating cost of an opened DC decreases with the number units of need satisfied by that depot.

Otherwise, the two models satisfy the same constraints. Equality (3.2) guarantees that each site will be linked to a unique predecessor by a single vehicle. Constraint (3.3) claims that the capacity of each vehicle will be respected. Relations (3.4) and (3.5) ensure the continuity of routes and that all vehicles travel back to the origin, i.e. the depot. Sub-tours are eliminated by Constraint (3.6). Inequality (3.7) means that only those locations selected as DCs can become the origin of a route. Constraint (3.8) indicates that the sum of sites' needs satisfied by a given DC cannot exceed the capacity of that depot. Constraints (3.9) - (3.11) show the binary nature of decision variables.

#### IV. RESOLUTION PROCESS

##### 4.1 Plausible Solutions Generation:

For our initial population as follows, we need to create a random feasible solution number. First, the algorithm develops a list of available depots, and another list of sites that are not yet served. Second, it randomly attributes a vehicle to any depot on the list, and



**Figure 6.1:** Simulated Annealing embedded within Genetic Algorithm

Then allocates sites to the nearest distribution center. Whenever a vehicle's capacity is exceeded, a new routing starts. If any DC's capacity get overloaded, that depot will be excluded from those available depots list. Any school already served would also be deleted from the list of sites that are not yet delivered. The whole algorithm stops as soon as that list has no more delivery sites. Then allocates schools to the nearest depot.

##### 4.2 Methods of Coding and Decoding:

The initial population is thereby found as in Sec. 4.3.1, yielding the feasible solutions. The algorithm then enters the next phase: coding each solution into a chromosome sequence.

Every single sequence quote one route, starting at a special depot and showing the order in which sites are served. Binary numbers are used to represent the depots from which the vehicles depart, and the schools that each vehicle serves. Each chromosome replies to one vehicle.

To be specific, binary number 0 means a certain school is not selected on this vehicle; contrary, we use 1 to represent those selected. Figure 6.2 shows an example with three potential depots and five sites. Here, depots D1 and D2 are opened. Thereby, the chromosomes indicated in Figure 6.3, corresponding to Figure 6.2, are explained as vehicle 1 departing from depot 1 delivering schools 2, 3, 5; and vehicle 2 departing from depot 2 serving sites 1 and 4

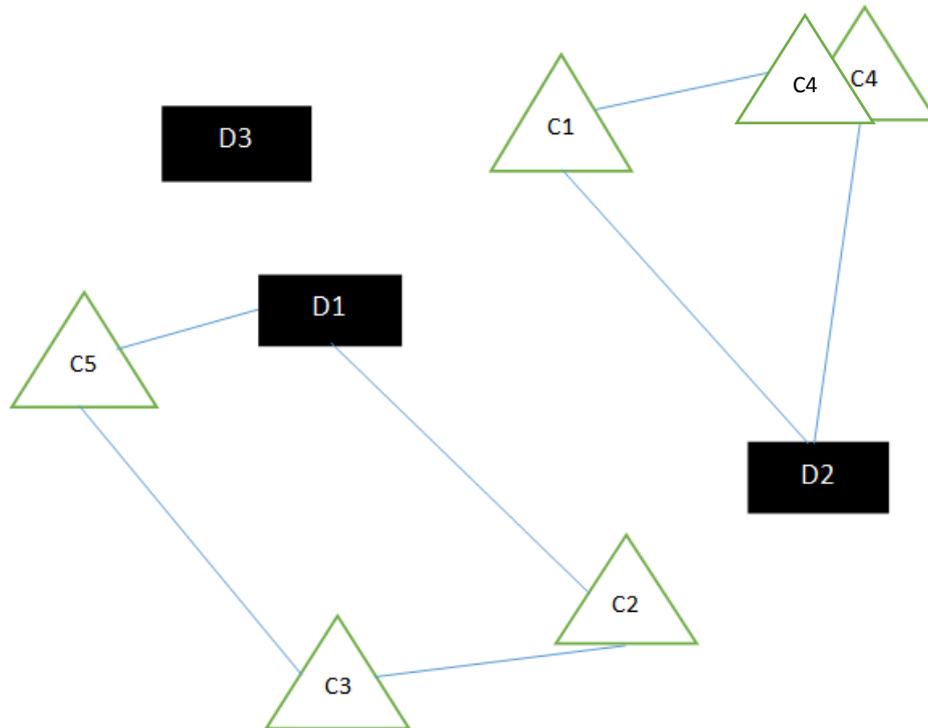


Figure 6.2: Sample customer assignments and routes

	Depot1	Depot2	Depot3	School1	School2	School3	School4	School5
Vehicle1	1	0	0	0	1	1	0	1
Vehicle2	0	1	0	1	0	0	1	0

Figure 4.3: Two sample chromosomes, corresponding to Figure 4.2

4.3 Filtered Calculation:

$$[FC] = \sum_{i \in I} A_i y_i + \sum_{j \in J} f_i \left( \sum_{j \in J} d_j v_{ij} \right) + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} C_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk} + \sum_{i \in I} P_1 \max \left( \sum_{j \in J} d_j v_{ij} - Q_i y_i, 0 \right) + \sum_{k \in K} P_2 \max \left( \sum_{i \in I} \sum_{j \in J} d_j x_{ijk} - W, 0 \right)$$

4.4 Inventory and Route Optimization

So every chromosomes are decoded for the simulated annealing process.

One generation to the next is pursued by a genetic algorithm. So, before let it to do so, we found that, in the genetic algorithm, it's easier to use simulated annealing, to expand the selection's probability for the next generation of the chromosomes that have a better fitness level. This employment of simulated annealing, in combination with a genetic algorithm was showed to be rewarding in a previous publication about the LIRP (Tuzkaya et al. 2012).

The principle of simulated annealing is followed by the probability of accepting the worse solution.

Simulated annealing is a probabilistic meta-heuristic that imitates the annealing process in metallurgy. The entire algorithm starts with an initial temperature and cooling-rate setting. The aim is to dodge a local optimum. The possibility to allow a worse solution is calculated by the formula:  $\exp\left(-\frac{\text{current cost} - \text{best cost}}{T}\right)$ . The gap between current routing cost and best cost and the temperature  $T$ , determines the chance to allow a worse solution. As we can see from this formula, if the gap between the two solutions is wider, to allow this inferior solution would be less likely. Likewise the temperature  $T$  becomes cooler (value becomes smaller) right after a certain amount of iterations. These pledges that the probability of allowing a worse solution will become smaller progressively. The lowest cost of each chromosome explored by this algorithm is returned as the "fitness" of that chromosome.

V. CALCULATION RESULTS

The performance of the proposed solution methods is now evaluated. The suggested Algorithms are coded in Matlab 2015b, and tested on a computer with a  $i-5$  Quad Core processor running at 2.6 GHz and 16 GB of memory.

In the experiments first group (Sec. 5.1), the demand points number  $n$  ranges from 10 to 100, with  $m = 3, 7, \text{ or } 15$  potential depots (DCs) opened. The transportation cost coefficient  $C_{ij}$  is 2, which means this cost is two times the distance traveled of Euclidean on arc  $(i; j)$ .

Depot coordinates and request points are generated from uniform distributions between  $[0, 50]$ . each vehicle capacity is 100. Making a depot acquires a random cost in the range of 800 to 1000. and the capacity  $Q_i$  of each depot is 450. the need of each site is produced uniformly in the range  $[20, 30]$ . About the experiments without economies case, we make the constant variable cost being \$8/ pallet. There we use 1 pallet of product as the need unit. When economies case is here, we use the function  $f_i$  in Eq. (3.1), whose initial slope (see Figure 3.1) much with \$8 /pallet. For all experiments, we make first the parameters of Eq. (3.1), after preliminary tests for the cases when economies case are here. Then, from the derivative of Eq. (3.1)

$f'(x) = \delta \gamma x^{\delta-1}$ , And, regarding to the average need  $\bar{x}$  in the particular data set, we set  $f'(\bar{x})$  as the initial slope

5.1 Simulated A. and Ant C.O. Comparison:

m	N	no economies-ACO				no economies-SA			
		number of DC opened	fixed cost	total cost	CPU(s)	number of DC opened	fixed cost	total cost	CPU(s)
3	10	1	0.86	3.26	5.86	1	0.86	3.26	5.85
	20	2	1.66	6.85	10.65	2	1.66	6.85	11.67
	30	2	1.66	8.42	19.16	2	1.66	8.54	19.37
7	30	2	1.76	8.54	22.0	2	1.76	8.56	23.2
	30	2	1.66	8.78	22.8	2	1.66	8.80	23.7
	50	4	3.50	12.0	50.3	4	3.48	12.2	52.9
	50	4	3.52	12.1	51.7	4	3.52	12.6	54.1
	70	4	3.52	15.8	97.0	4	3.52	15.9	101.1
	70	4	3.52	15.6	96.1	4	3.52	15.6	100.3
15	100	6	5.28	18.9	307.7	6	5.30	19.2	309.9
	100	7	6.10	20.8	309.8	7	6.10	21.1	310.2

Table 7.1: Algorithm comparison: No economies All costs in \$000

m	n	with economies-ACO				with economies-SA			
		number of DC opened	fixed cost	total cost	CPU(s)	number of DC opened	fixed cost	total cost	CPU(s)
3	10	1	0.86	2.68	5.01	1	0.86	2.68	5.00
	20	2	1.66	5.43	12.28	2	1.66	5.43	13.23
	30	2	1.66	7.17	18.98	2	1.66	7.28	19.37
7	30	2	1.76	7.26	24.7	2	1.76	7.29	25.6
	30	2	1.66	7.24	25.2	2	1.66	7.28	26.6
	50	3	2.73	10.3	50.1	3	2.53	10.4	52.9
	50	3	2.53	9.98	49.2	3	2.53	10.4	52.0
	70	4	3.52	13.6	96.0	4	3.56	13.9	101.4
	70	4	3.52	13.5	96.8	4	3.52	13.7	102.7
15	100	5	4.38	16.0	308.2	5	4.40	16.4	311.7
	100	5	4.42	16.8	310.0	5	4.42	16.9	315.5

Table 7.2: Algorithm comparison: With economies of scale all costs in \$000

The results returned by the genetic algorithm with ant colony optimization are compared by these Tables 7.1 and 7.2, to those from simulated annealing containing an embedded genetic algorithm.

In those tables, we use  $\gamma = 25$  and  $\delta = 2/3$  (See Eq. 3). the model in Table 7.1 has no economies case. Both tables show that for the small-size instance (3 DCs and 10 or 20 sites), both algorithms give the same result. Like, when there are no economies case, a single depot is opened with a fixed cost of \$860 and total cost of \$3,260 for that moment including 3 depots and 10 sites. Running times are almost the same for the two approaches.

Detours start to show as the size of the pattern grows. From Table 7.1, consider the first range when  $m = 7$  and  $n = 50$ . All both methods recommend four depots, but since the resulting fixed costs differ, we know that the methods open several depots. Though the ACO has a fixed cost of \$3,500, \$200 greater than that of SA, it has a lower total cost of \$12,000.

Besides the running time isn't enough for ACO. For the same patterns but for the model with economies case, we got similar results in Table 7.2. Generally, from the cost and running time perspectives, the genetic algorithm with ACO performs better than SA embedded within a genetic algorithm in these experiments.

We are suspecting but can't prove that the additional running time of simulated annealing over ant colony optimization may become greater as the problem size instances increase. Thereby, we use ACO with the genetic algorithm to test additional instances and analyze the economies case influence.

## VI. CONCLUSIONS

In this paper, a location Inventory-routing problem with variable cost of depots is considered. The problem minimizes the total cost, including fixed cost of setting depots, operating cost which changes with the transportation cost and output. Here we present two objective functions. All both regard operating cost of depots but, one of the accounts for distribution center economies case. Here those concave costs are represented by using a power function  $\gamma X^\delta$ , where  $X$  denotes the facility output.

Since the LRP combines two NP-hard problems (facility location problem and vehicle routing problem), most of papers use heuristics. Therefore we consider two metaheuristic methods: simulated annealing embedded within a genetic algorithm, and a genetic algorithm with ant colony optimization. Through preliminary trial, we decide employing the genetic algorithm with ant colony optimization for more experiments. Firstly we take away the variable cost part to make a comparison between the other published methods and performance. Using publicly-available data sets, the results says that our proposed method is acceptable. The median gaps, relative to best results known are lower than 3% in Tables 5.3 and 5.5

Then, with those data sets, we make a comparison of economies case with variable cost, to those when economies case are present. We saw that fewer distribution centers are opened and total costs are less for the cases with economies case. Furthermore, we studied the exponent influence  $\delta$  in the power function representing the economies case. Firstly we test how the total cost got affected by the power value  $\delta$ . We try three values of  $\delta$ , and give mathematical proof, both showing that with a raise in  $\delta$ , the operating cost rises and there is less impact of economies case. Next, we studied the tradeoff between transportation cost and operating cost. From former experiments, we remark that when economies case are present less distribution centers  $s$  are opened, therefrom the traveling distances are wider. Economies case output total costs that are less though the transportation costs are wider and the required amount of vehicles will not reduce.

Changing some parameters, we make the certain instances total cost approximately equal to display the transaction. We found that, without raising the total cost, the spares got through by economies case authorize extra cost on transportation. It may enable the distribution centers equipping heighten service to schools.

Taking reckoning the operating costs helps to accomplish solutions near to reality.

An amount of research itinerary can be studied in coming times. Firstly, for unceasing research, for LRP variable cost can be regarded as a part of the objective function. Although the problem had no need to discuss economies case, adding the operating cost of depots or warehouses has practical value.

Secondly, the model can be stretched. Sources supplier, i.e. firms can be added to the current model. Even more, economies case can be taken into account for the two-echelon LRP, which regards the product from flow firms to depots to schools (site), and routing are developed at both levels. We can introduce variable cost into the two-echelon LRP to deceive real-world situations in the distribution system of a supply chain.

Furthermore solution methods can get better. Though results says that our proposed method may be allowed, the gap compared to the best known result (BKR) is not small for some particular pattern. Actually most of research employs heuristic approaches, even on the standard LRP.

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